ICAMOP journal, vol. 1, no. 1, 2019, p. 2-7
www.icamop.periodikos.com.br

A Stochastic Production and Maintenance Planning Optimisation for Multi Parallel Machine

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Abstract - Our manufacturing system is composed of m multiple parallel machines (m> 1) manufacturing one type of product. Indeed, the production system receives a random request during a finite time horizon of which a level of service is required. The demand is greater than the total capacity of the machine park, hence the need for outsourcing to compensate for the missing demand. In addition, we consider that the unit cost of subcontracting is variable depending on the quantity subcontracted. The main objective of this approach is to jointly optimize for each period the production rate, the subcontracting rate, the preventive maintenance rate and the working time of each machine, while minimizing the costs of production, inventory holding, backlog, outsourcing and maintenance.

Keywords: Production, maintenance, multi parallel machine, optimization, stochastic, degradation.

1. Introduction

Outsourcing has become increasingly important in the manufacturing industry, it has become a necessity for coordination, improvement and cooperation, in order to satisfy the customer's need in terms of quantity, cost and time. Moreover, in view of the dependence and interaction between production and maintenance and the need for outsourcing, very little research has been done in the sense of joint management of production and maintenance under constraints of sub-contracting. In addition, the case of a production system subject to stochastic deterioration is studied by **[1].** In their work, they evaluated this degradation, proposing a maintenance model that considers both corrective and preventive maintenance. In the same vein, the reference [2] dealt with the problem of joint production, maintenance and control of emissions, including the potentially degrading production system. The aim of their work is to simultaneously control the production rate, the emission rate and the maintenance rate in order to mitigate the effects of system degradation by minimizing the total cost over an infinite horizon.

Until now, it has been discussed one machine. Several other configurations exist. Among these, we can note the parallel configuration. The latter is found in the factory where several machines perform the same operation at a specific time of the process, in order to increase the production rate. This is noted by [3], who have studied a manufacturing system subject to failure consisting of several identical machines and each machine produces a single type of product. In the case of parallel machines, the integration of preventive maintenance and tactical production planning for a production system consisting of a set of parallel components is the work objective of [4]. Additionally, [5] studied the case of several parallel systems that produce a single type of product.

2. Description of problem and hypotheses

2.1. Motivations and research objectives

The production policies of manufacturing systems are generally influenced by uncertainties such as random demand, variation in availability. Hence the need to improve relations between companies in the sense of collaboration and cooperation to reduce these uncertainties.

The industrialization situation considered consists of two companies, the DO company and the subcontracting company. The DO enterprise consists of a system of production of m parallel-mounted machines that produce a single type of manufacturing product. The DO receives random requests over time d_k characterized by a normal distribution with a first and a second statistic. Demand d_k exceeds the combined maximum production capacity of the Umax machine fleet. To meet this demand, the DO uses outsourcing to compensate for the missing demand. In this paper we study the problem of integrated management of maintenance and production in case of a complex system multi-machines. Indeed, parallel machines are degradable as a function of time and production rate. Therefore, we consider the influence of the production rate on the degradation of the manufacturing system, to determine the production rate of each machine, the rate of preventive maintenance of each machine and finally the quantity outsourced. Certainly, our model is characterized by: multi-machines, degradation of the production line, random demand, service level, variation of the production rate, variation of preventive maintenance rate and variation of the subcontracting rate.

2.2. Hypotheses

Before establishing the analytical model, it is necessary to present the following assumptions:

- We study the case of a synchronous and homogeneous production system.
- The production system is subject to a degradation process with a growing rate.
- We consider a dependence between the rate of production and the rate of failure.
- Degradation is influenced by the rate of production.
- All machines are selected for production and no machine is idle during production.
- The parallel machines of the production line are identical: example the processing time of the product per unit time.
- The subcontracting rate is considered variable and depends on production capacity as well as customer demand.
- The failure of all machines in the production line is detected instantly.
- The working time of the machines in the production line is variable.
- The installation cost compared to the change in production rate is negligible.
- We assume that the repair and replacement times are negligible.
- The store S stores the products manufactured by the company DO and the outsourcing company and they are delivered directly to the customer with each request.
- The cost of delivery of manufactured products is negligible.

-Storage, production and maintenance costs; C^+, C^- , C_{pro}, C_{cm} and C_{pm} are known and constant.

- The unit cost of subcontracting ***Csub*** is variable depending on the quantity subcontracted.
- The demand is random and is characterized by the standard deviation of demand σ^2_{dk} and the average demand $\hat{d}_k$ for each period k are known and constant.

3.Mathematical model

3.1. Production policy

In our proposed model, the total expected cost includes production, maintenance, outsourcing, holding and backlog costs. We give the horizon manufacturing horizon **H** constitutes **N** homogeneous periods of duration **Δt**.

3.1.1. Total quantity of production of the parallel line

Of course, for any production line, it is necessary to determine the total quantity of production (Eq.1). We consider that all the machines in the production line are identical, but they do not have the same rate of production. Indeed, the total quantity of production of the parallel line is the sum of the normal working rates of the set of parallel machines during the period k multiplied by the identical production rate of all machines is given by equation 2.

U_k (u, t_{ik})= (1)

$(u = (u1, u2, \ldots, ui, um) \geq 0, tik \geq 0)$

Uk=u$\sum_{i=1}^{m} t_{ik}$ for i ϵ (i=1, …, m) et k ϵ (k=1, …, N) (2)

Where we distinguish the following elements:

m: Number of machines mounted in parallel with m> 1,

U_k: Parallel line production rate during each period k,

tik: Normal working time of parallel line machines during each period k,

u: Same production rate for all machines,

3.1.2. Inventory level during each period

The inventory stock level of the x_k store during period k, represents the inventory level at period k equals the stock level at period k-1 plus the sum of the combined production rate and the production rate subcontractor during period k, minus demand d_k during the same period k. Therefore, the inventory level at period k is given by the following equation:

$$x_k = x_{k-1} + \Delta t\,(U_k + V_{k,l}) - d_k \quad (3)$$

3.1.3. Production cost

The cost of production CP represents the cost of production of the manufacturing company. The latter is obtained by multiplying the unit production cost **Cpro** by the sum of the total quantity of production of the parallel line $\mathbf{U_k}$ during each period k and the length of the production period Δt during the period k along the horizon H. Δt (Eq.4).

$$CP\,(U_k) = Cpro\ \Delta t \sum_{k=1}^{N} U_k \quad (4)$$

3.1.4. Subcontracting cost

We consider the dependence between the subcontracting rate or the outsourcing rate and the unit cost of subcontracting [6] and [7] (Eq.5). The DO always looks for a subcontractor with minimal cost. Therefore, the subcontractor offers several price lists [6].

$$\text{Csub} = \begin{cases} Csub_1 & \text{si} \quad 0 < V\,k,l \le Vk,1 \\ \quad \dots & \\ Csub_l & \text{si} \quad V\,k,l-1 < V\,k,l \le V\,k,l \\ \quad \dots & \\ Csub_L & \text{si} \quad VL-1 < V\,k,l \le \bar{v} \end{cases}$$

(5)

With : $V_{k,l} = \alpha.\, d$ et $0 < \alpha \le 1$ et $\bar{v} = d$

$$Csub_1 < \dots < Csub_l < \dots < Csub_L$$

The rank of subcontracting prices (l =1, …, L),
Therefore, the total OC subcontracting cost can be expressed as follows:

$$OC = Csub_l\ \Delta t \sum_{k=1}^{N} V_{k,l} \qquad (6)$$

3.1.5.Inventory holding cost

The inventory holding cost is for each positive inventory unit remaining at the end of a period. Holding cost includes the inventory cost of each period. It is given by the following equation:

$$IC = C^{+}.x^{+} \qquad (7)$$

$$\text{With: } x^{+} = \max(0, x) = \sum_{k=1}^{N} x_k \qquad (8)$$

3.1.6. Backlog cost

Backlog cost represents orders produced late or not executed, because of accumulation of service or accumulation of an order produced, which makes it impossible to meet the deadline and the delay of delivery. We assume a delay rather than a loss of sales (backlogging) if demand at a given period cannot be satisfied. Therefore, we now assume that x_k can take positive or negative values, which in the other case represents inventory holding. In addition, a unit penalty cost **C-** is charged for each unit late in each period (Eq.9).

$$BC = C^{-}.(-x)^{+} \qquad (9)$$

$$\text{Avec : } (-x)^{+} = \sum_{k=1}^{N} \max(-x_k, 0) \qquad (10)$$

3.2. Maintenance strategy

The maintenance strategy adopted concerns only the park machines of the DO company, because the subcontracting company manages the maintenance of its equipment by itself. The figure below shows the manufacturing system maintenance strategy considered.

We consider that:

-The maintenance strategy for all parallel machines of the manufacturing company is a preventive maintenance with minimal repair on the fine horizon H.

-The PM preventive maintenance actions are supposed to be perfect, the parallel machines are then considered new (as good as new) after each PM.

-When a machine breaks down between two successive revisions (PM) a minimum repair is performed (CM).

- We consider that the failure rate λ is continuous and cumulative.

-Minimal repair restores the equipment to a new state (As good as new).

-Influence of the variation of the rate of production in the degradation of the machine.

-The duration of corrective and preventive maintenance actions is negligible.

-Optimization of maintenance consists in determining the fractions of horizon H during which the preventive maintenance actions must be performed.

-The time interval [0, H] is partitioned into $\boldsymbol{\theta}$ parts of duration ***T***.

-A preventive maintenance action is performed ***n.Ti*** (n = 1, 2, ... θ) and (i = 1, ..., M) with T> Δt.

-We consider that $\boldsymbol{\theta_i}$ is the number of preventive maintenance actions during horizon H.

-We consider that the relation between $\boldsymbol{\theta_i}$ and ***Ti*** is given by:

$$H = T_i \times \theta_i \qquad (11)$$

Admittedly, optimization of the strategy is characterized by the optimal number of preventive maintenance action $\boldsymbol{\theta^*}$ preventive maintenance actions to be performed for each machine on the finished horizon H. However, in order to calculate the maintenance cost, we assume that the ***Cpm*** and ***Ccm*** costs incurred by the preventive and corrective maintenance actions respectively are known and constant, with ***Ccm*** >> ***Cpm***. In addition, we assume that the repair and replacement times are negligible. The expression of the maintenance cost CM given by equation 12 is based on the model proposed by de Hajej, Turki and Rezg [***8***].

$$MC\,(U_i, \theta_i) = Cpm\,.(\theta_i - 1) + Ccm.\, A_i(U_i, \theta_i) \qquad (12)$$

for i ϵ (i=1, …, m)

With:

-MC (U_i, θ_i): Maintenance strategy cost.

-$A_i(U_i, \theta_i)$: The average number of failures for each machine during the planning horizon ***H*** according to the production plan for each.

- θ_i : The number of preventive maintenance actions during horizon ***H***.

Indeed, the probability of degradation of a machine is described by the probability density function of the time at failure ***f (t)*** and for which the failure rate ***λ (t)*** increases with time and the rate of production ***u(t)*** (Eq.13). That is why; a preventive maintenance action is programmed according to the production rate in order to reduce the breakdown of the machine.

$$\lambda(t)=\frac{f(t)}{[1-F(t)]} \qquad (13)$$

In addition, we assume that machine degradation is linear, and the failure rate increases with time and production rates. We consider that the failure rate λ is continuous and cumulative. The final default rate is the sum of the default rates for each period. This expresses the influence of the rate of production on the degradation of the manufacturing system [**9**]. In addition, the function of the failure rate $\lambda_{ik}(t)$ is linear. Then, the nominal failure rate λ_{in} ***(t)*** corresponding to a maximum production rate. We assume that machine degradation is linear and that the failure rate increases with time and level of production (Eq.14).

$$\lambda_{ik}(t) = \lambda_{i(k-1)}(\Delta t) + \frac{U_{ik}}{Umax}\lambda_{in}(t) \qquad \forall\, t \in [0, \Delta t] \quad (14)$$

This expression can be expressed as follows:

$$\lambda_{ik}(t) = \lambda_0 + \sum_{p=1}^{k-1}\frac{U_{ip}}{Umax}\,\lambda_{in}(t) + \frac{U_{ik}}{Umax}\lambda_{in}(t) \quad (15)$$

$\forall\, t \in [0, \Delta t]$

With : $\lambda_0(t_0) = \lambda_0$

System reliability is described by the failure law characterizing the nominal conditions. It is represented by the Weibull law defined by β scale parameter, α shape parameter and γ position parameter. In addition, the initial failure rate is represented by:

$$\lambda_{in}(t) = \frac{\alpha}{\beta} \times \left(\frac{t-\gamma}{\beta}\right)^{\alpha-1} \qquad (16)$$

The average number of failures for each machine according to the production plan of each machine is given by:

$$Aj = \sum_{i=1}^{j} \int_0^{\Delta t} \lambda_{ik}(t)\,dt \qquad (17)$$

$$A_i(U_i, \theta_i) = \qquad (18)$$

$$\sum_{j=0}^{\left(\frac{(H-1)}{T}-1\right)} \left(\sum_{i=(jT+1)}^{(j+1)T} \sum_{i=1}^{m} \int_0^{\Delta t} \lambda ik(t)\,dt\right) + \sum_{k=\left(\frac{(H-1)}{T}\right).T+1}^{H} \sum_{i=1}^{m} \int_0^{\Delta t} \lambda ik(t)\,dt$$

4.Integrated control policy of production and maintenance

The determination of the production plan amounts to developing and minimizing the total cost relative to the two production systems. This problem can be formulated as an optimal control problem, linear and stochastic under a service level constraint, whose objective is to find the minimum value of the objective function under certain constraints.

In fact, the total cost per unit of time for our integrated production and maintenance plan includes:

- The cost of production: **CP**
- Inventory holding cost: **IC**
- Backlog cost: **BC**
- Subcontracting cost: **OC**
- Maintenance cost: **MC**

The stochastic linear total cost can be formulated by the following expression:

Min C :

$$\sum_{k=1}^{N} \left[Cpro\,\Delta\tau_k\, U_k + Csub_l\,\Delta t\, V_{k,l} + C^{+}.x^{+} + C^{-}.(-x)^{+} + \sum_{i=1}^{Mk}(Ccm\; C_i(U_i, \theta_i) + Cpm(\theta_i - 1))\right]$$

Subject to:

$x_k = x_{k-1} + \Delta t\,(U_k + V_k) - d_k$ (20)

$U_{min} \le U_k \le U_{max}$ (21)

$U_{max} < d_k$ (22)

$0 \le V_{k,l} \le V_{max}$ (23)

$\theta_{min} \le \theta_i \le \theta_{max}$ (24)

$1 \le t_{ik} \le t_{max}$ (25)

Prob $(\hat{x}_k \ge 0) \ge a$ (26)

$U_k, \theta_i, t_{ik}, V_k \ge 0$ (27)

$\forall$ (k=1, …, N) , (i=1, …, m) et (ℓ=1, …, L),

5. Optimization of integrated production control and maintenance plan using outsourcing: numerical example

We assume in our model that a company is made up of three machines in parallel. They manufacture a single type of product, the demand for which is variable for each period. We note that um = monetary unit and ut = time unit. The planning horizon of the production plan is equal to 10 months (H = 10). The main data of the problem are presented in the following table:

Tableau 1. Numerical data of the problem

m	U_{max}	U_{min}	V_{max}	t_{ik}^{max}	t_{ik}^{min}	Cpro	U
3	18	6	15	1	3	7	2
Ccm	**Cpm**	**C+**	**C-**	**β**	**α**	**γ**	**Δt**
3000	500	0.25	43	100	2	0	1

Tableau 2. Average demand

k	1	2	3	4	5	6	7	8	9	10
d_k	22	22	20	21	19	18	20	19	19	20

We used the fmincon optimization method to solve the problem. We repeated the test 10 times and found that the optimal total cost value is 6761.3um. In addition, Figure 1 shows the value of the optimal cost function as a function of iteration number.

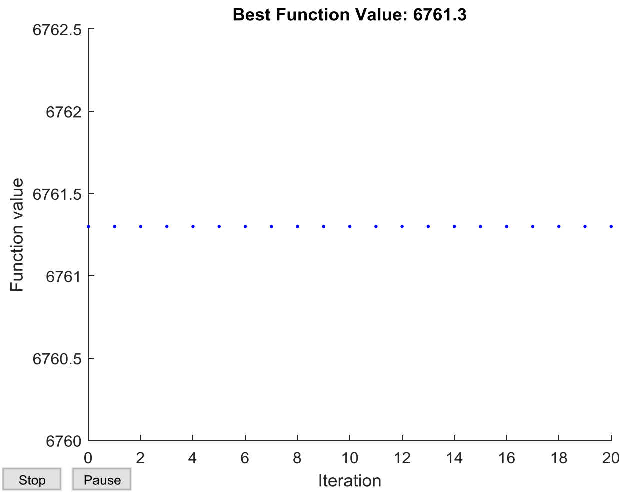

Figure 1. The value of the optimal cost function according to number of iterations

Tableau 3. Optimum values

Periods	Average Demand	Decision variables				Costs for 3 machines in parallel				
		Production rate	Sub-contracting rate	Inventory level	PM Rate	Production	Sub-contracting	Inventory holding	Backlog	Maintenance
Period 1	22	14	13	0						
Period 2	22	12	14	4						
Period 3	20	16	2	2						
Period 4	21	12	14	7						
Period 5	19	18	10	16	(3,2,4)	910	2822	24.5	0	3004.8
Period 6	18	16	1	15						
Period 7	20	12	4	11						
Period 8	19	6	8	6						
Period 9	19	14	15	16						
Period 10	20	10	15	21						
Total cost					6761.3um					

Tableau 4. Machine working time and their preventive maintenance rates

	Taux_MP	**Period 1**	**Period 2**	**Period 3**	**Period 4**	**Period 5**	**Period 6**	**Period 7**	**Period 8**	**Period 9**	**Period 10**
Machine 1	3	1	2	2	3	3	3	2	1	1	1
Machine 2	2	3	3	3	2	3	3	1	1	3	3
Machine 3	4	3	1	3	1	3	2	3	1	3	1

5. Conclusion and Perspectives

In this paper, we have developed the stochastic optimization model of the problem of controlling the production and maintenance of an identical three-machine manufacturing system in parallel, under constraint of subcontracting. Considering the decision variables (production rate, outsourcing rate, preventive maintenance rate of each machine and the working time of each machine), minimizing the costs of production, inventory holding, backlog, subcontracting and maintenance. For a variable demand and a level of service required. First, we have formulated a linear stochastic problem giving an optimal control policy. Further research is needed to establish the extension of the proposed model to the case of manufacturing systems involving several products and machines, considering the quality level among the selection criteria of the subcontractors.

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