



# Experimental study of the first mode shape for determining the amplitude-frequency dependence of a CSCS rectangular plate

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**Abstract—** The paper describes an experimental study of Clamped, Simply-supported, Clamped, and Simply-supported (CSCS) elastic thin rectangular plates subject to a large amplitudes forced vibration. Our goal is to determine the amplitude-frequency dependence. For this end, a test rig was designed and made, on which an electrodynamic exciter located at the center of the plate provides the transverse harmonic excitation whereas an accelerometer measure the vibration amplitudes generated. The experimental data will compared to numerical result obtained and published by Z. Beidouri et al. We have also demonstrate the feasibility of the Benamar's method, already experimentally validated in fully clamped rectangular plates made by different materials, in the CSCS boundary conditions case.

**Keywords—** response dynamic; amplitude frequency dependence; first mode shape; numerical experimental analysis

## I. INTRODUCTION:

The linear theory of structural dynamic behavioral analysis is now well established in engineering for massive structures [1], despite the inherent structural nonlinearities.

In addition, for modern engineering the structures must be efficient and low weight. This type of structures is used in the area of defense, aerospace and others and aims to develop new theories of reliable and optimal structures that allow designing efficient and safe structures.

To understand large structures, we must understand the elements that contribute to their construction, for example: plates, beams, shells, etc, these structures easily deform by friction with air or by the force transmitted as reported in [3]. This phenomenon generates excitations around the resonance frequencies and to deal with it, many researchers have developed new mathematical models that undertake the geometric nonlinearity of these structures that cannot be neglected any more [2].

In the literature, several theories have been proposed to explain the nonlinearity geometric concerning the thin elastic structures like plates. To solve this issue, many researchers have proposed various methods as Benamar's model discussed in [4] and [5].

The Benamar's model [4-6], which is based on Hamilton's principle and spectral analysis, allows calculating the contribution coefficient of the higher modes in order to determine the nonlinear frequencies and nonlinear mode shape at large vibration amplitudes [7-11]. In the last few years, Z. Beidouri et al. have demonstrated numerically the validity of Benamar's method in the case of C-S-C-S and C-S-S-S boundary condition in [7].

This paper aims to experimentally valid the previously published numerical results on amplitude-frequency dependence in the case of rectangular plates with C-S-C-S boundary conditions [7].



II. EXPERIMENTAL DETAILS :

To study the geometric nonlinearity of a rectangular plate, the theoretical model presented in [7] was adopted. We summarize below the mechanical and geometrical characteristics of the RP tested:

- Material: Aluminum alloy.
- Young Modulus (E):  $7 \times 10^{10} \text{ N/m}^2$ .
- Density ( $\rho$ ):  $2410 \text{ kg/m}^3$ .
- Poisson modulus ( $\nu$ ): 0.346.
- Length (a): 240mm.
- With (b): 360mm.
- Thickness (H): 0.6mm.
- Aspect Ratio ( $\alpha=a/b$ ): 0.66.

Figure1 illustrates the test rig, built at Superior School of Technology Casablanca (ESTC, Morocco) and used in this investigation, and shows a homogenous Rectangular Plate (RP) fixed at its four boundaries to be isolated from the external vibrations. An electrodynamic exciter is located at the center of the RP, and provides transverse harmonic excitation, and the accelerometer (Brüel&Kjær 10mV/g) is used to measure the amplitude vibrations generated. In our study, the imperfections in the boundary conditions due to fabrication and material are neglected.

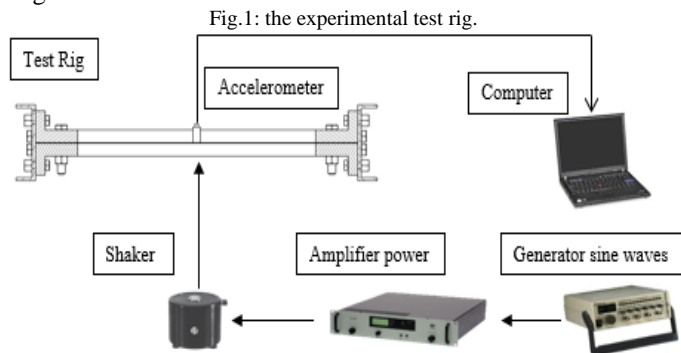


Fig.1: the experimental test rig.

A. Free Vibration Analysis:

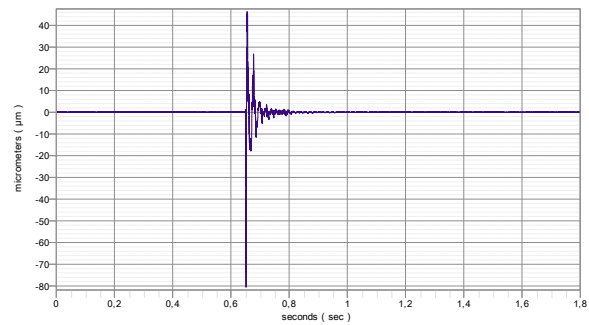
This section is devoted to some experimental results obtained on the rectangular plate mentioned in the above section, which allow highlighting a certain number of nonlinear phenomena, which are at the heart of the following studies.

During a vigorous strike at the center of the plate, which vibrates in 3.16 seconds, we can notice that the amplitude signal that measured by the accelerometer in the center of the rectangular plate evolves over time, the amplitude decreases to cease after 0.3 second due to damping. Figure2 shows a spectrogram obtained during a free analysis of the rectangular plate, which represents the evolution of the frequency of the amplitude of a point of the structure as a function of time.

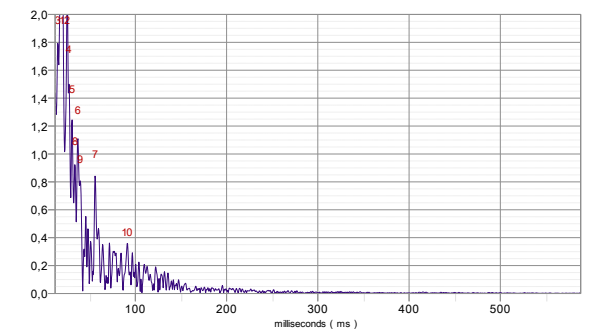
From free vibration analysis, we conclude that structure is non-linear, and in front of the complexity to determine all nonlinear dynamic parameters by this study, due to excite many mode shapes simultaneously. Forced vibration analysis, which will be presented in the following section used to study

the nonlinear dynamic behavior qualitatively observed, the plate is excited via an electrodynamic exciter (shaker).

Fig.2: Free vibration test (a) displacement response of vigorous strike, (b) spectrogram evolution of the amplitude in 1 second of rectangular plate



(a)

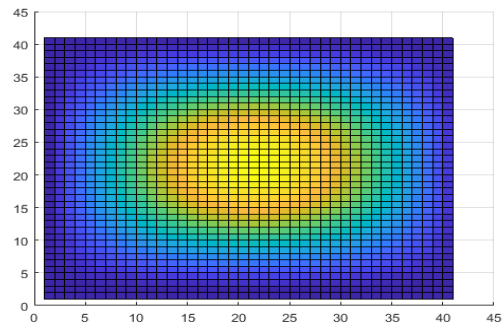


(b)

B. Forced vibration Analysis:

The rectangular plate mentioned above, is excited harmonically via an electrodynamic exciter around the resonance frequency, which identified numerically fig.3(a) referring to [7] also the mode shape is illustrated in fig.3(b), which obtained experimentally by exciting the plate around the resonance frequency with fixed amplitude, and using the sand on the plate's surface. Figure3 shows the deflection of the first mode shape, which lead us to know the right position of the maximum vibration amplitude.

Fig.3: identification the coordination of the maximum amplitude associated to the first mode shape: (a) numerical results given in reference [7], (b) experimental



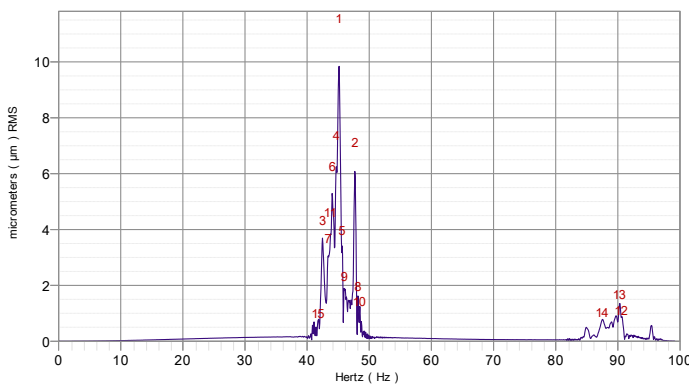
(a)



(b)

The accelerometer is glued on the plate's center in order to measure the maximum amplitude of the first mode shape. The excitation frequency is varied gradually near the fundamental frequency while the amplitude vibration maintained less than plate's thickness in order to measure the peak amplitude corresponding to the first mode shape. Fig.4 shows the steady state periodic vibration associated to the first mode shape, the result obtained show that experimental resonance frequency equal to 45Hertz is less than the linear fundamental frequency, which is equal to 49 Hz, obtained from the numerical result [7]. This difference comes from both the added masses, which are glued, to the plate, the first is from the accelerometer and the second is from the coil of the dynamic exciter.

Fig.4: resonance frequency associated to the first mode shape

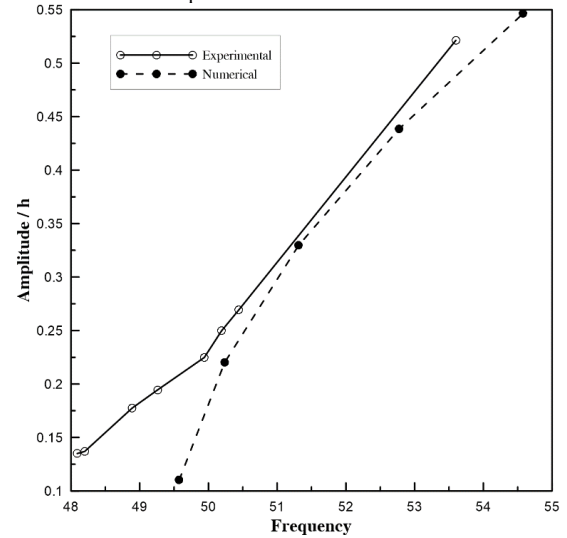


### III. AMPLITUDE FREQUENCY DEPENDENCE:

Figure5 illustrate two curves of frequencies and normalized amplitudes that obtained numerically in the article [7], and experimental results by exciting the rectangular plate with a sinusoidal force in the center with keeping its amplitude constant and varying gradually the excitation frequency around the first resonance. The amplitude frequency dependence is illustrated on the experimental and numerical curves, this phenomenon called geometric nonlinearity.

Observing that response of frequency is less than linear frequency due to added masses get it through the glued coil of shaker at the plate [5] and the mass accelerometer that used to measure the forced vibration.

fig.5: backbone curves of the first mode shape, numerical results [7], experimental measurement.



### IV. CONCLUSION:

So far, we have studied the amplitude frequency dependence of the first mode shape experimentally in the case of C-S-C-S rectangular plate and compared with the results obtained in [7]. The test rig is presented in the previous section that lead us to investigate the nonlinearity geometric at large vibration amplitudes of rectangular plate with ratio equal to 0.66 and 0.6mm in thickness. The free vibration response of C-S-C-S rectangular isotropic plates have investigated using Benamar's model.

From the experimental pattern of results carried out, it can be concluded that the research into the amplitude frequency dependence of the rectangular plate is similar to that obtained on free vibration investigation referring to [7]. The difference between the fundamental and the resonance frequency comes from the added masses of the coil and accelerometer. On the basis of the promising findings presented in this paper, work on the remaining issues is continuing and will be presented in future papers.

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