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Numerical modeling of dam-break problems 1-D on dry bed

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***Abstract*— This paper is concerned with the application and comparison of a finite element Taylor Galerkin and finite volume method for the simulation of shallow water flows to model dam break problems on dray bed. The finite volume scheme uses Roe's approximate Riemann solver to evaluate the convection terms combined with the MUSCL technique to achieve second-order accuracy in space. The finite element model is based on the Lax-Wendroff tow-step scheme, which is second-order in space and time. The performance and efficiency of the tow algorithms are illustrated and compared through dam break problems.**

Keywords*— Dam break; shallow water equations; finite element; Lax-Wendroff scheme; Finite volume; Roe's scheme;MUSCL technique

I. INTRODUCTION (*Heading 1*)

There are about 45 000 dams in the word, for hydropower, water supply and irrigation, or the regulation of rivers. Any hydraulic structure can undergo security failures that can lead to more accidents; one can save an average of about three annual dam failures. For calculations in civil engineering and design, we need to identify the behavior of the propagation of the flood wave, and give answers to how the volume of water released propagates in the water stream. The purpose in the present work is to investigate two numerical schemes: the Lax-Wendroff scheme with two steps in the finite element version and the finite volume Roe scheme, and make comparisons through one dimensional shallow-water equations. We propose in this study the dam breaking in a river.

The paper is organized as follows. After written Saint-Venant equations in section 2, we present in section 3 the formulation of the finite element method. In section 4 the scheme of Roe in finite volume method is detailed. Section 5 is devoted to numerical results. Finally, section 6 contains some conclusions.

II. ONE-DIMENSIONAL EQUATIONS TO BE SOLVED:

The set of governing Saint-Venant equations in the 1D case is the following:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = R_s \tag{1}$$

U is unknowns vectors, F flux vectors, R_s source term.

$$U = \begin{Bmatrix} h \\ hu \end{Bmatrix} \; ; \; F = \begin{Bmatrix} hu \\ hu^2 + \frac{1}{2} g h^2 \end{Bmatrix} \tag{2}$$

Where h is the water depth, u the velocity component, in the present work we consider equation (1) without source and diffusion terms. The model of Saint-Venant is established while applying the principles of conservations of the mass and the quantity of movement while respecting some hypotheses:

1) Hydrostatic distribution of the pressure one notes that this hypothesis is not more valid in problems off strong curvatures of the free surfaces.
2) The speed is considered constant according to the vertical.

III. FINITE ELEMENT LWR SCHEME:

The numerical solution is computed in two steep by using Taylor series expansion in the time step Δt:

$$\begin{cases} \mathrm{U}^{\mathrm{n}+\frac{1}{2}} = \mathrm{U}^{\mathrm{n}} + \frac{1}{2}\Delta\mathrm{t}\frac{\partial \mathrm{U}^{\mathrm{n}}}{\partial \mathrm{t}} + 0(\Delta\mathrm{t}^2) \\ \mathrm{U}^{\mathrm{n}+1} = \mathrm{U}^{\mathrm{n}} + \Delta\mathrm{t}\frac{\partial \mathrm{U}^{\mathrm{n}+\frac{1}{2}}}{\partial \mathrm{t}} + 0(\Delta\mathrm{t}^2) \end{cases} \quad (3)$$

The finite elements model of the above expression is gotten while writing the weak formulation:

$$\begin{cases} W_I = \int_{x_0}^{x_N} P_e \left(U^{n+\frac{1}{2}} - U^n\right) dx + \frac{1}{2}\Delta t \int_{x_0}^{x_N} P_e \frac{\partial F^n}{\partial x} dx \\ \qquad - \frac{1}{2}\Delta t \int_{x_0}^{x_N} P_e {R_s}^n dx \\ W_{II} = \int_{x_0}^{x_N} \psi(U^{n+1} - U^n) dx - \Delta t \int_{x_0}^{x_N} \frac{\partial \psi}{\partial x} F^{n+\frac{1}{2}} dx \\ \qquad - \Delta t \int_{x_0}^{x_N} \psi {R_s}^n dx + \Delta t [F\psi]_{x_0}^{x_N} \end{cases} \quad (4)$$

ψ is the test unction, the admissible approximation spaces for the integral form are Boulerhcha (1995):

$$\begin{cases} U^n = N_j {U_j}^n + N_{j+1} {U_{j+1}}^n \\ F^n = N_j {F_j}^n + N_{j+1} {F_{j+1}}^n \\ {R_s}^n = N_j {R_j}^n + N_{j+1} {R_{j+1}}^n \end{cases} \quad (5)$$

ψ is the test function identical to the weight functions N is the test function identical to the weight functions Nj, Pe is tests function constant on every element, Pe=1 on the considered element and hopeless elsewhere. The spaces of admissible approximation for the integrate shape are linear. $\mathrm{U}^{\mathrm{n}+\frac{1}{2}}$ is constant on every element, Ni is expressed in the plan of reference Oξ. For a linear discretization:

$$\begin{cases} N_j = \frac{1-\xi}{2} \\ N_{j+1} = \frac{1+\xi}{2} \end{cases} \quad (6)$$

We write WII under the following shape:

$$W_{II} = \sum_e W^e \quad (7)$$

We is the elementary residual that writes under the shape:

$$W^e = \langle\psi\rangle([m^e]\{\Delta U\} - \{r^e\}) \quad (8)$$

After assembly of the elementary residues we will have to solve the following system:

$$[M]\{\Delta U\} = \{R\} \quad (9)$$

This scheme is explicit, the criteria of stability is defines by the condition of the Current Friedricks-Lewys Boushaba (2008):

$$\Delta t \leq min\left(\frac{\Delta x}{\left(u + \sqrt{gh}\right)_{max}}\right) \quad (10)$$

IV. FINITE VOLUME ROE SCHEME:

In the finite volume formulation, the computational domain is first discretized into finite number of control volumes. The weak formulation is then obtained by integrating the system of equations over each control volume, which leads generally to the evaluation of discrete fluxes over the boundaries of each control volume Alcrudo (1993).

We shall present only mean stages of the scheme. This integration of the Saint6Venant equation is done over a finite volume T_i, the application of Green formula leads to:

$$\Delta x \frac{\partial U}{\partial t} + \sum_{j \epsilon E(i)} \int_{\Gamma_{ij}} F n_x d\Gamma = 0 \quad (11)$$

We denote by Γij the interface between two cells, E(i) is the set of elements that have a common point with elements xi, the problem is to evaluate the convection flux $\mathcal{F}(\mathrm{U}, \vec{\mathrm{n}}) = \mathrm{Fn}_{\mathrm{x}}$ over the three borders off the cell, we seek an approximation of:

$$\int_{x+1/2} \mathcal{F}(U) d\Gamma = \Phi(U_i, U_{i+1}) \quad (12)$$

In order to construct a scheme taking into account these flow directions, an appropriate decomposition of the flux related to positive and negative propagation speeds is needed. Roe (1981) proposed a particular approximate Riemann solver based upon the use of local linearized Jacobian matrices. The numerical flux is written as follows:

$$\Phi(U_l, U_r) = \frac{1}{2}\left(\mathcal{F}(U_l) + \mathcal{F}(U_r)\right) - \frac{1}{2}\left|\tilde{\mathcal{A}}(U_l, U_r)\right|(U_l - U_r) \quad (13)$$

Where $\tilde{\mathcal{A}}$ is a constant matrix at every time level and for each pair of states Ul and Ur. $\tilde{\mathcal{A}}$ must verify the following conditions:

- Conservation of the scheme
- The consistency with the original problem
- Property of hyperbolicity

V. NUMERICAL RESULTS:

We present some model test case proposed in the literature (see, among others Fennema (1990); Brufau (2003)). The dam will simply consist of a body of water which is maintained at a constant level upstream and downstream by a salve as shown in Fig.1. It is assumed that the bottom is flat and we are interested in what happens on the axis (OZ). The problem is modeled using the one-dimensional Saint-Venant equations without source term.

Computational domain is rectangular channel 2m long, discretized with 200 points. Initially 1m of water depth is used upstream the dam and 0m of depth downstream:

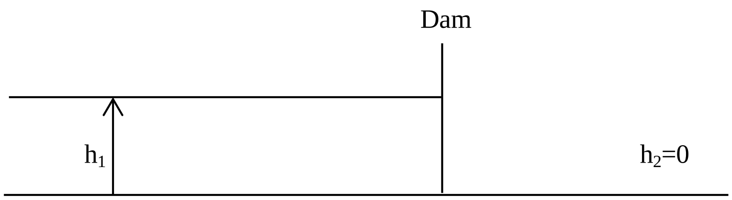

Figure 1 Initial condition for the 1D dam-break problem.

$$h(x,t=0) = \begin{cases} h_L = 1m \; if \; x \le 0 \\ h_R = 0 \; if \; x > 0 \end{cases} \tag{14}$$

and u(x,t=0)=0 for all x ∈ I

For all the computations presented here, the courant number CFL was set to 0.7 in order to ensure the stability condition.

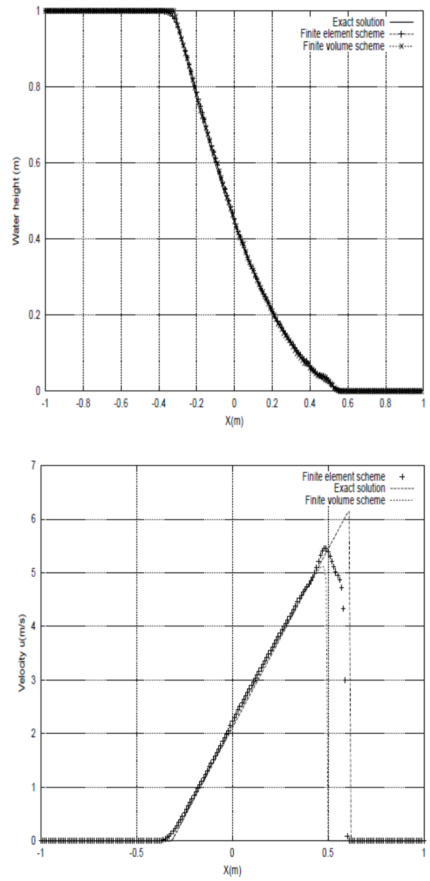

Figure 2 1D cross-section on h and u at time t=0.15 sec.

Fig.2 presents the water depth and velocity profiles at time t=0.15s, using the two methods. A good agreement can be seen between the finite element Taylor-Galerkin and the finite volume Roe scheme compared with the exact solution. We can see also that there is a slight diffusion created by the finite element scheme: the shock is spread on a number of nodes greater than the Roe scheme. This diffusion is mainly due to the introduction off the dissipation term in the finite element method in order to attenuate the oscillations of the Taylor-Galerkin scheme. We can also notice that the Taylor Galerkin scheme gives more accurate results at the velocity profile to the vicinity of the hydraulic jump.

For a quantitative comparison, we resent in table (1) the absolute L_2 error norms off the solutions obtained by the finite volume Roe scheme and the finite element Taylor-Galerkin scheme. The results are presented for the water height and water velocity at time t=0.15s. It is clear that errors obtained by the Roe scheme are low compared to those by Taylor-Galerkin scheme. The numerical solution by the Roe's scheme is then more accurate for this test case. However, in terms of efficiency, an evaluation of the CPU time for both schemes at the physical time 0.15s is:

Scheme	$\left\Vert h_{ext} - h_{app} \right\Vert_2$	$\left\Vert u_{ext} - u_{app} \right\Vert_2$
Roe scheme	0.0021	0.0335
Galerkin scheme	0.0033	0.0031

Table1. L_2 error norms of the water height and water velocity for the dam break over dry bed at time t=0.15s.

$$CPU(EFT - G) = 1.4s$$
$$CPU(V\ F\ ROE) = 3s \tag{15}$$

Which means that the Taylor-Galerkin method requires 2 times less computational work than the Roe's scheme. This is due to the fact that the Roe scheme evaluates the Jacobian matrix on each interface of the element, which is expansive in time of computation.

CONCLUSION

In this article, we simulated and compared the dam break problem on wet dry with two different methods: a second order centered scheme written in finite element context, and a second order upwind Roe scheme in finite volume formulation. Similar results and good agreement are found between the two approaches for the wet bottom test. For this test case, it is also estimated that the finite volume scheme is more accurate, less diffusive but more consuming in CPU time. Concerning the dry bed test, the Taylor Galerkin finite element seems to be more accurate without the use of the diffusion term, a parameter which is generally difficult to adjust.

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